Reconstruction of lossless compressed fingerprint and x-ray images using Haar matrix technique

Reem Talib Miri, Hussain Ali Hussain

Abstract— **The objective of this research is to decompress an image using steps not by one step to get an image with dimension and size different than the original without lossless any part of image the number of steps to reconstruction an image depend on the image dimensions. If the image dimensions are 512×512 then it needs 9 steps to decompress. And if it was 8×8 then it will need only 3 steps as explained in paper.**

Index Terms—scaling function, Haar matrix, Haar wavelet, Lossless compression.

I. INTRODUCTION

From an historical point of view, wavelet analysis is a new method, though its mathematical underpinnings date back to the work of Joseph Fourier in the nineteenth century. Fourier laid the foundations with his theories of frequency analysis, which proved to be enormously important and influential. The attention of researchers gradually turned from frequency-based analysis to scale-based analysis when it started to become clear that an approach measuring average fluctuations at different scales might prove less sensitive to noise. The first recorded mention of what we now call a "wavelet" seems to be in 1909,in a thesis by Alfred Haar. The concept of wavelets in its present theoretical form was first proposed by Jean Morlet and the team at the Marseille Theoretical Physics Center working under Alex Grossman in France. The methods of wavelet analysis have been developed mainly by Y.Meyer and his colleagues, who have ensured the methods' dissemination. The main algorithm dates back to the work of Stephane Mallat in 1988. Since then, research on wavelets has become international. Such research is particularly active in the United States, where it is spearheaded by the work of scientists such as Ingrid Daubechies, Ronald Coifman, and Victor Wickerhauser. Barbara Burke Hubbard describes the birth, the history, and the seminal concepts in a very clear text. See *"The World According to Wavelets,"* A.K.Peters, Wellesley, 1996.The wavelet domain is growing up very quickly. A lot of mathematical papers and practical trials are published every month [2-3-4].

II. HAAR WAVELETS OPERATIONAL MATRIX

We shall begin with the more convenient way for representing Haar wavelets in computer and for $x \in$[A,B] which was given by [1] and for this purpose we define the quantity $M=2^J$, where J is the maximal level of resolution and

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divide the interval [A,B] into 2M subintervals of equal length; each subinterval has the length.

$$\Delta x = (B - A)/2M.$$

Two parameters are introduced the dilation parameter j for which $j= 0,1,...,J$ and the translation parameter $k=0,1,...,m-1$ where $m = 2^j$. The wavelets number i is identified as $i=m+k+1$ the ith Haar wavelet is defined as:

$$h_i(x) = \begin{cases} 1, & \text{for } x \in [\xi_1(i), \xi_2(i)], \\ -1, & \text{for } x \in [\xi_2(i), \xi_3(i)], \\ 0, & \text{elsewhere.} \end{cases} \quad \cdots \quad (1)$$

Where:

$$\xi_1(i) = A + 2k\mu\Delta x,$$
$$\xi_2(i) = A + (2k+1)\mu\Delta x,$$
$$\xi_3(i) = A + 2(k+1)\mu\Delta x,$$
$$\mu = M/m$$

The case $i=1$ corresponding to the scaling function

$$h_1(x) = \begin{cases} 1, & \text{for } x \in [A, B] \\ 0, & \text{elsewhere.} \end{cases} \quad \cdots \quad (2)$$

Taking the collocation points as following

$$x_i = A + (i - 0.5)\Delta x, \quad i = 1,2,...,2M \quad \cdots \quad (3)$$

And Haar vector $H_{2M}(x_i)$ are defined as:

$$H_{2M}(x_i) = [h_1(x_i), h_2(x_i),..., h_{2M}(x_i)]^T \quad \cdots \quad (4)$$

By letting A=0, B=1 (usually the Haar wavelets are defined for the interval $x \in [0,1]$) and hence $\Delta x = \frac{1}{2M}$ in eq.(3).We define the 2M-square Haar matrix $H_{2M \times 2M}$ as:

$$H_{2M \times 2M} = [H_{2M}(\tfrac{1}{4M}) H_{2M}(\tfrac{3}{4M}) ... H_{2M}(\tfrac{4M-1}{4M})] \cdots \quad (5)$$

Let $J=2$ where J is the maximal level of resolution and divide the interval [0,1] in to 8 subintervals of equal length; each subinterval has the length $\Delta x = 1/8$.

Two parameters are introduced the dilation parameter j for which $j=0,1,2$ and the translation parameter $k=0$ when $m=1$ and $k=0,1$ when $m=2$ and $k=0,1,2,3$ when $m=4$. The wavelets number i is defined as $i=m+k+1$.

i	2	3	4	5	6	7	8
m	1	2	2	4	4	4	4
k	0	0	1	0	1	2	3

For $i=2$, $m=1$, $k=0$
$\mu=4$, $\Delta x = 1/8$
$\xi_1(2) = 0$, $\xi_2(2) = 1/2$, $\xi_3(2) = 1$

$$h_2(x) = \begin{cases} 1 & x \in [0, \tfrac{1}{2}], \\ -1 & x \in [\tfrac{1}{2}, 1], \\ 0 & \text{else where.} \end{cases}$$

For $i=3$, $m=2$, $k=0$
$\mu=2$, $\Delta x = 1/8$

$\xi_1(3) = 0$, $\xi_2(3) = 1/4$, $\xi_3(3) = 1/2$

$$h_3(x) = \begin{cases} 1 & x \in \left[0, \frac{1}{4}\right], \\ -1 & x \in \left[\frac{1}{4}, \frac{1}{2}\right], \\ 0 & else\ where. \end{cases}$$

For $i=4$, $m=2$, $k=1$

$\mu = 2$, $\Delta x = 1/8$

$\xi_1(4) = 1/2$, $\xi_2(4) = 3/4$, $\xi_3(4) = 1$

$$h_4(x) = \begin{cases} 1 & x \in \left[\frac{1}{2}, \frac{3}{4}\right], \\ -1 & x \in \left[\frac{3}{4}, 1\right], \\ 0 & else\ where. \end{cases}$$

For $i=5$, $m=4$, $k=0$

$\mu = 2$, $\Delta x = 1/8$

$\xi_1(5) = 0$, $\xi_2(5) = 1/8$, $\xi_3(5) = 1/4$

$$h_5(x) = \begin{cases} 1 & x \in \left[0, \frac{1}{8}\right], \\ -1 & x \in \left[\frac{1}{8}, \frac{1}{4}\right], \\ 0 & else\ where. \end{cases}$$

For $i=6$, $m=4$, $k=1$

$\mu = 2$, $\Delta x = 1/8$

$\xi_1(6) = 1/4$, $\xi_2(6) = 3/8$, $\xi_3(6) = 1/2$

$$h_6(x) = \begin{cases} 1 & x \in \left[\frac{1}{4}, \frac{3}{8}\right], \\ -1 & x \in \left[\frac{3}{8}, \frac{1}{2}\right], \\ 0 & else\ where. \end{cases}$$

For $i=7$, $m=4$, $k=2$

$\mu = 1$, $\Delta x = 1/8$

$\xi_1(7) = 1/2$, $\xi_2(7) = 5/8$, $\xi_3(7) = 6/8 = 3/4$

$$h_7(x) = \begin{cases} 1 & x \in \left[\frac{1}{2}, \frac{5}{8}\right], \\ -1 & x \in \left[\frac{5}{8}, \frac{6}{8}\right], \\ 0 & else\ where. \end{cases}$$

For $i=8$, $m=4$, $k=3$

$\mu = 1$, $\Delta x = 1/8$

$\xi_1(8) = 3/4$, $\xi_2(8) = 7/8$, $\xi_3(8) = 1$

$$h_8(x) = \begin{cases} 1 & x \in \left[\frac{3}{4}, \frac{7}{8}\right], \\ -1 & x \in \left[\frac{7}{8}, 1\right], \\ 0 & else\ where. \end{cases}$$

Taking the collocation points as following:

$x_1 = 1/16$, $x_2 = 3/16$, $x_3 = 5/16$, $x_4 = 7/16$,

$x_5 = 9/16$, $x_6 = 11/16$, $x_7 = 13/16$, $x_8 = 15/6$

Haar vector $H_8(x_i)$ are defined as:

$H_8(x_i) = [\ h_1(x_i)\ ,\ h_2(x_i)\ ,\ h_3(x_i)\ ,\ h_4(x_i)\ ,\ h_5(x_i)\ ,\ h_6(x_i)\ ,\ h_7(x_i)\ ,\ h_8(x_i)\ ]^T$

For the first Haar vector

$H_8(\frac{1}{16}) = [\ h_1(\frac{1}{16})\ ,\ h_2(\frac{1}{16})\ ,\ h_3(\frac{1}{16})\ ,\ h_4(\frac{1}{16})\ ,\ h_5(\frac{1}{16})\ ,\ h_6(\frac{1}{16})\ ,\ h_7(\frac{1}{16})\ ,\ h_8(\frac{1}{16})\ ]^T$

$H_8(\frac{1}{16}) = [\ 1\ ,\ 1\ ,\ 1\ ,\ 0\ ,\ 1\ ,\ 0\ ,\ 0\ ,\ 0\ ]^T$

For the second Haar vector

$H_8(\frac{3}{16}) = [\ h_1(\frac{3}{16})\ ,\ h_2(\frac{3}{16})\ ,\ h_3(\frac{3}{16})\ ,\ h_4(\frac{3}{16})\ ,\ h_5(\frac{3}{16})\ ,\ h_6(\frac{3}{16})\ ,\ h_7(\frac{3}{16})\ ,\ h_8(\frac{3}{16})\ ]^T$

$H_8(\frac{3}{16}) = [\ 1\ ,\ 1\ ,\ 1\ ,\ 0\ ,\ -1\ ,\ 0\ ,\ 0\ ,\ 0\ ]^T$

For the third Haar vector

$H_8(\frac{5}{16}) = [\ h_1(\frac{5}{16})\ ,\ h_2(\frac{5}{16})\ ,\ h_3(\frac{5}{16})\ ,\ h_4(\frac{5}{16})\ ,\ h_5(\frac{5}{16})\ ,\ h_6(\frac{5}{16})\ ,\ h_7(\frac{5}{16})\ ,\ h_8(\frac{5}{16})\ ]^T$

$H_8(\frac{5}{16}) = [\ 1\ ,\ 1\ ,\ -1\ ,\ 0\ ,\ 0\ ,\ 1\ ,\ 0\ ,\ 0\ ]^T$

For the fourth Haar vector

$H_8(\frac{7}{16}) = [\ h_1(\frac{7}{16})\ ,\ h_2(\frac{7}{16})\ ,\ h_3(\frac{7}{16})\ ,\ h_4(\frac{7}{16})\ ,\ h_5(\frac{7}{16})\ ,\ h_6(\frac{7}{16})\ ,\ h_7(\frac{7}{16})\ ,\ h_8(\frac{7}{16})\ ]^T$

$H_8(\frac{7}{16}) = [\ 1\ ,\ 1\ ,\ -1\ ,\ 0\ ,\ 0\ ,\ -1\ ,\ 0\ ,\ 0\ ]^T$

For the fifth Haar vector

$H_8(\frac{9}{16}) = [\ h_1(\frac{9}{16})\ ,\ h_2(\frac{9}{16})\ ,\ h_3(\frac{9}{16})\ ,\ h_4(\frac{9}{16})\ ,\ h_5(\frac{9}{16})\ ,\ h_6(\frac{9}{16})\ ,\ h_7(\frac{9}{16})\ ,\ h_8(\frac{9}{16})\ ]^T$

$H_8(\frac{9}{16}) = [\ 1\ ,\ -1\ ,\ 0\ ,\ 1\ ,\ 0\ ,\ 0\ ,\ 1\ ,\ 0\ ]^T$

For the sixth Haar vector

$H_8(\frac{11}{16}) = [\ h_1(\frac{11}{16})\ ,\ h_2(\frac{11}{16})\ ,\ h_3(\frac{11}{16})\ ,\ h_4(\frac{11}{16})\ ,\ h_5(\frac{11}{16})\ ,\ h_6(\frac{11}{16})\ ,\ h_7(\frac{11}{16})\ ,\ h_8(\frac{11}{16})\ ]^T$

$H_8(\frac{11}{16}) = [\ 1\ ,\ -1\ ,\ 0\ ,\ 1\ ,\ 0\ ,\ 0\ ,\ -1\ ,\ 0\ ]^T$

For the seventh Haar vector

$H_8(\frac{13}{16}) = [\ h_1(\frac{13}{16})\ ,\ h_2(\frac{13}{16})\ ,\ h_3(\frac{13}{16})\ ,\ h_4(\frac{13}{16})\ ,\ h_5(\frac{13}{16})\ ,\ h_6(\frac{13}{16})\ ,\ h_7(\frac{13}{16})\ ,\ h_8(\frac{13}{16})\ ]^T$

$H_8(\frac{13}{16}) = [\ 1\ ,\ -1\ ,\ 0\ ,\ -1\ ,\ 0\ ,\ 0\ ,\ 0\ ,\ 1\ ]^T$

For the eighth Haar vector

$H_8(\frac{15}{16}) = [\ h_1(\frac{15}{16})\ ,\ h_2(\frac{15}{16})\ ,\ h_3(\frac{15}{16})\ ,\ h_4(\frac{15}{16})\ ,\ h_5(\frac{15}{16})\ ,\ h_6(\frac{15}{16})\ ,\ h_7(\frac{15}{16})\ ,\ h_8(\frac{15}{16})\ ]^T$

$H_8(\frac{15}{16}) = [\ 1\ ,\ -1\ ,\ 0\ ,\ -1\ ,\ 0\ ,\ 0\ ,\ 0\ ,\ -1\ ]^T$

We define the 8-square Haar matrix $Ħ_{8\times8}$ as:

$Ħ_{8\times8} = [\ H_8(\frac{1}{16})\ \ H_8(\frac{3}{16})\ \ H_8(\frac{5}{16})\ \ H_8(\frac{7}{16})\ \ H_8(\frac{9}{16})\ \ H_8(\frac{11}{16})\ \ H_8(\frac{13}{16})\ \ H_8(\frac{15}{16})\ ]$

$$Ħ_{8\times8} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \\ 1 & 1 & -1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & -1 & -1 \\ 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \end{bmatrix}$$

The Haar matrix is the multiplication of the following three matrices. $Ħ_{8\times8} = ħ_3 . ħ_2 . ħ_1$

Where $ħ_1$, $ħ_2$, $ħ_3$ are:

$$\hbar_1=\begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \end{bmatrix}$$

$$\hbar_2=\begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\hbar_3=\begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$C=\begin{bmatrix} 123.8281 & -13.0469 & 1.2813 & 4.1875 & 20.9375 & -5.625 & 10.1875 & -3.5625 \\ 7.0156 & 6.8281 & -5.6563 & 11.25 & -2.6875 & -0.125 & -4.6875 & 4.9375 \\ -1.8438 & 6.8438 & -3.75 & -0.3125 & 6.25 & -3 & 2.5 & 29.875 \\ 3.5 & 2.8125 & 0.8125 & -5.8125 & -7.125 & -8 & 15.125 & -9.75 \\ 1.625 & 0.75 & 2.125 & -13.375 & 5.5 & 0.75 & -21 & 0.25 \\ 8.3125 & 1.5625 & -7.375 & 8.75 & -7.5 & -22.75 & -14 & 2.5 \\ -3.6875 & 4.6875 & 5 & 9.875 & -7.5 & -2 & -2 & 2.25 \\ -3.4375 & 3.3125 & -2.625 & -15.5 & 7.25 & 7.5 & 9.75 & -19.25 \end{bmatrix}$$
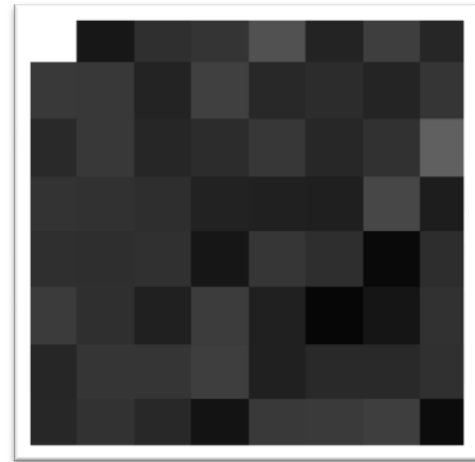

Figure (2) represents the matrix C

III. STEPS FOR RECONSTRUCTION GRAYSCALE IMAGE

We will explain process on 8×8 image matrix. The process can be generalized to $2^j \times 2^j$ matrix, where is j natural number. If there are 2^j elements in row and column in matrix, then the process of row and column will consist of j steps. In our case j =3. Let A is an image matrix given below by using the command (imread image) in MATLAB.
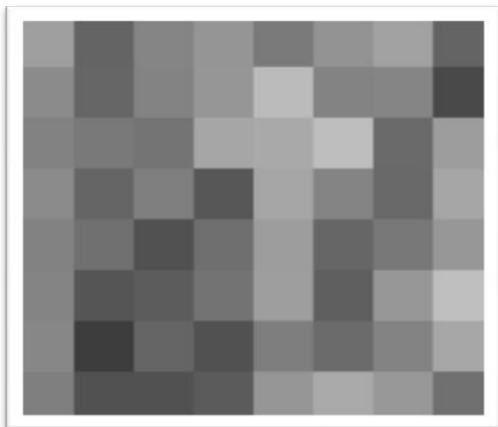

Figure (1) represents the matrix A

$$A=\begin{bmatrix} 156 & 96 & 130 & 146 & 118 & 144 & 159 & 96 \\ 136 & 98 & 128 & 147 & 185 & 127 & 130 & 68 \\ 126 & 117 & 112 & 163 & 166 & 188 & 102 & 154 \\ 136 & 97 & 123 & 83 & 163 & 129 & 101 & 163 \\ 126 & 108 & 76 & 107 & 154 & 98 & 116 & 148 \\ 129 & 81 & 88 & 111 & 155 & 91 & 148 & 189 \\ 132 & 56 & 97 & 77 & 122 & 103 & 128 & 164 \\ 123 & 76 & 77 & 87 & 147 & 167 & 149 & 108 \end{bmatrix}$$

And C is an image compression matrix

First step for reconstruction

The product of $(\hbar_3)^T \times C \times \hbar_3 = RS_1$ is equivalent to the following procedures. For every columns of C we calculate the addition of, and subtraction between, the first two elements. And replace the original numbers with the above results. We apply the same procedure to all other columns to get matrix B_1. Then for every rows of B_1 we calculate the addition of, and subtraction between, the first two elements. And replace the original numbers with the above results. We apply the same procedure to all other rows to get matrix RS_1.

$$RS_1=\begin{bmatrix} 124.625 & 137.0625 & -4.375 & 15.4375 & 18.25 & -5.75 & 5.5 & 1.375 \\ 96.9375 & 136.6875 & 6.9375 & -7.0625 & 23.625 & -5.5 & 14.875 & -8.5 \\ 5 & -8.6875 & -3.75 & -0.3125 & 6.25 & -3 & 2.5 & 29.875 \\ 6.3125 & 0.6875 & 0.8125 & -5.8125 & -7.125 & -8 & 15.125 & -9.75 \\ 2.375 & 0.875 & 2.125 & -13.375 & 5.5 & 0.75 & -21 & 0.25 \\ 9.875 & 6.75 & -7.375 & 8.75 & -7.5 & -22.75 & -14 & 2.5 \\ 1 & -8.375 & 5 & 9.875 & -7.5 & -2 & -2 & 2.25 \\ -0.125 & -6.75 & -2.625 & -15.5 & 7.25 & 7.5 & 9.75 & -19.25 \end{bmatrix}$$

Figure (3) represents the matrix RS_1

Second step for reconstruction

The product of $(\hbar_2)^T \times RS_1 \times \hbar_2 = RS_2$ is equivalent to the following procedures. For every columns of RS_1 we calculate the addition of the first and third elements and their subtraction. We also calculate the addition of the second and fourth elements and their subtraction. And replace the original numbers with the above results. We apply the same procedure to all other columns to get matrix B_2. Then for every rows of B_2 we calculate the addition of the first and third elements and their subtraction. We also calculate the addition of the second and fourth elements and their subtraction. We repeat the same procedure for all rows to obtain matrix RS_2.

$$RS_2 = \begin{bmatrix}
121.5 & 137.75 & 143.5 & 113.25 & 24.5 & -8.75 & 8 & 31.25 \\
119 & 120.25 & 161.5 & 130 & 12 & -2.75 & 3 & -28.5 \\
111 & 95.5 & 124.5 & 150.25 & 16.5 & -13.5 & 30 & -18.25 \\
96.75 & 84.5 & 134.75 & 137.25 & 30.75 & 2.5 & -0.25 & 1.25 \\
4.5 & 0.25 & -12.5 & 14.25 & 5.5 & 0.75 & -21 & 0.25 \\
2.5 & 17.25 & 15.5 & -2 & -7.5 & -22.75 & -14 & 2.5 \\
6 & -4 & 1.5 & -18.25 & -7.5 & -2 & -2 & 2.25 \\
-2.75 & 2.5 & -22.25 & 8.75 & 7.25 & 7.5 & 9.75 & -19.25
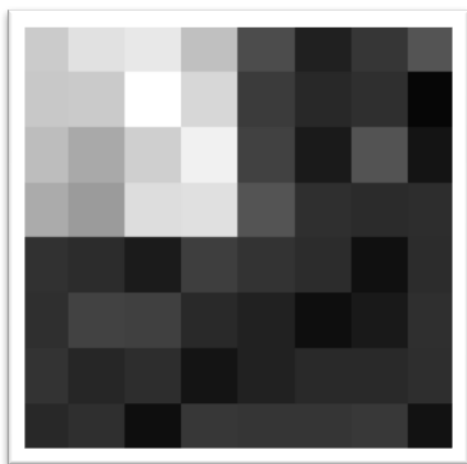\end{bmatrix}$$

Figure (4) represents the matrix RS_2

Third step for reconstruction

The product of $(\hbar_1)^T \times RS_2 \times \hbar_1 = RS_3 = A$ is equivalent to the following procedures. For every columns of RS_2 we calculate the addition and subtraction of, first and fifth elements, Second and sixth elements, third and seventh elements, fourth and eighth elements. To get new matrix B_3 Then for every rows of B_3 we calculate the addition and subtraction of first and fifth elements, second and sixth elements, third and seventh elements, fourth and eighth elements. To get matrix $RS_3 = A$.

$$RS_3 = A = \begin{bmatrix}
156 & 96 & 130 & 146 & 118 & 144 & 159 & 96 \\
136 & 98 & 128 & 147 & 185 & 127 & 130 & 68 \\
126 & 117 & 112 & 163 & 166 & 188 & 102 & 154 \\
136 & 97 & 123 & 83 & 163 & 129 & 101 & 163 \\
126 & 108 & 76 & 107 & 154 & 98 & 116 & 148 \\
129 & 81 & 88 & 111 & 155 & 91 & 148 & 189 \\
132 & 56 & 97 & 77 & 122 & 103 & 128 & 164 \\
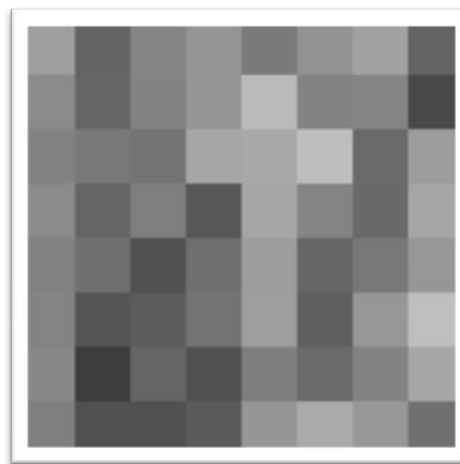123 & 76 & 77 & 87 & 147 & 167 & 149 & 108
\end{bmatrix}$$

Figure (5) represents the matrix RS_3 equal to A

We considered a small matrix because it is difficult to show the results on the large size of matrices like a matrix of size 512×512. Now we use fingerprint or x-ray image of size $512 \times 512 = 2^9 \times 2^9$. First we will extend the inverse Haar matrix up to the size of the fingerprint or x-ray image. Later, we will apply the inverse Haar matrix to the image. The whole procedure will be carried out with the help of MATLAB. In our case the image matrix is of the dimension $512 \times 512 = 2^9 \times 2^9$, so inverse Haar matrix method compressed the image of fingerprint or x-ray in ninth steps the first step compresses to $256 \times 256 = 2^8 \times 2^8$ the second step compresses to $128 \times 128 = 2^7 \times 2^7$ the third step compresses to $64 \times 64 = 2^6 \times 2^6$ the fourth step compresses to $32 \times 32 = 2^5 \times 2^5$ the fifth step compresses to $16 \times 16 = 2^4 \times 2^4$ the sixth step compresses to $8 \times 8 = 2^3 \times 2^3$ the seventh step compresses to $4 \times 4 = 2^2 \times 2^2$ the eighth step compresses to $2 \times 2 = 2^1 \times 2^1$ the ninth step compresses to $1 \times 1 = 2^0 \times 2^0$ then we can sent the image by mail in internet or load it in website to decompressed the image after downloaded from the internet. We will used Haar matrix to reconstruction image in ninth steps the first step reconstructs to $2 \times 2 = 2^1 \times 2^1$ the second step reconstructs to $4 \times 4 = 2^2 \times 2^2$ the third step reconstructs to $8 \times 8 = 2^3 \times 2^3$ the fourth step reconstructs to $16 \times 16 = 2^4 \times 2^4$ the fifth step reconstructs to $32 \times 32 = 2^5 \times 2^5$ the sixth step reconstructs to $64 \times 64 = 2^6 \times 2^6$ the seventh step reconstructs to $128 \times 128 = 2^7 \times 2^7$ the eighth step reconstructs to $256 \times 256 = 2^8 \times 2^8$ the ninth step reconstructs to $512 \times 512 = 2^9 \times 2^9$ and we have the image.

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